Discrete Quantum Group	Discrete Group
$S = c_0\text{-}\bigoplus_{\alpha\in R} B(H_\alpha)$, H_α fd $\alpha : S \to B(H_\alpha)$, $p_\alpha = \mathrm{id}_{H_\alpha} \in S$, $p_\alpha S = B(H_\alpha)$ $\mathcal{S} = \text{alg-}\bigoplus_{\alpha\in R} B(H_\alpha)$	$S = c_0(\Gamma)$, $R = \Gamma$, $H_\alpha = \mathbb{C}$ $\alpha = ev_\alpha$, $p_\alpha = 1\!\!1_\alpha$ $\mathcal{S} = \mathbb{C}^{(\Gamma)}$
$\delta : S \to M(S \otimes S)$ coassociative, $\kappa : \mathcal{S} \to \mathcal{S}$ $\varepsilon : S \to \mathbb{C}$ co-unit (trivial repr. : $\varepsilon \in R$)	$\delta(a)(\alpha, \beta) = a(\alpha\beta)$ $\varepsilon(a) = a(e)$, $\kappa(a)(\alpha) = a(\alpha^{-1})$
Haar weights h_L, h_R defined on $\mathcal{S}$ $\forall\ a \in p_\alpha S\ h_L(a) = m_\alpha \mathrm{Tr}\ (F_\alpha^{-1} a)$ and $h_R(a) = m_\alpha \mathrm{Tr}\ (F_\alpha a)$ with $F_\alpha \in B(H_\alpha)_+$ st $\mathrm{Tr}\ F_\alpha = \mathrm{Tr}\ F_\alpha^{-1} =: m_\alpha$	$h_L = h_R$ counting measure on Γ (S, δ) unimodular : $h_L = h_R \Leftrightarrow \forall\alpha$ $F_\alpha = \mathrm{id}_{H_\alpha} \Leftrightarrow \kappa^2 = 1 \Leftrightarrow \hat{h}$ tracial
$\Lambda : \mathcal{S} \to H$ GNS construction for h_R $V(\Lambda \otimes \Lambda)(x \otimes y) = (\Lambda \otimes \Lambda)(\delta(x)(1 \otimes y))$ $V \in M(\hat{S}_r \otimes S)$ with $\hat{S}_r = (\mathrm{id} \otimes B(H)_*)(V)^-$	$H = \ell^2(\Gamma)$, $H \otimes H \simeq \ell^2(\Gamma \times \Gamma)$ $V(\xi)(\alpha, \beta) = \xi(\alpha\beta, \beta)$ $V = \sum \rho_\alpha \otimes 1\!\!1_\alpha \in M(C_r^*(\Gamma) \otimes c_0(\Gamma))$
$\mathcal{F}(a) = (\mathrm{id} \otimes h_R)(V^*(1 \otimes a)) \in \hat{S}_r$ for $a \in \mathcal{S}$ $\hat{\mathcal{S}} = \mathcal{F}(\mathcal{S}) \subset \hat{S}_r$, $\hat{h}(\mathcal{F}(a)^* \mathcal{F}(a)) = h_R(a^* a)$	$\mathcal{F}(a) = \sum a(\alpha) \rho_{\alpha^{-1}}$, $\hat{\mathcal{S}} = \mathbb{C}\Gamma \subset C_r^*(\Gamma)$

In the case of an abelian discrete group, $C_r^*(\Gamma) \simeq C(\hat{\Gamma})$ via $\alpha \mapsto \langle \cdot, \alpha \rangle$. Then

$$V = [(\hat{\alpha}, \alpha) \mapsto \langle \hat{\alpha}, \alpha \rangle] \in C_b(\hat{\Gamma} \times \Gamma) \quad \text{and} \quad F(a) = [\hat{\alpha} \mapsto \textstyle\sum_\alpha a(\alpha) \langle \hat{\alpha}, \alpha^{-1} \rangle].$$

Discrete Quantum Group	Compact Group
$S = c_0\text{-}\bigoplus_{\alpha\in R} B(H_\alpha)$, H_α fd $\alpha : S \to B(H_\alpha)$, $p_\alpha = \mathrm{id}_{H_\alpha} \in S$, $p_\alpha S = B(H_\alpha)$ $\mathcal{S} = \text{alg-}\bigoplus_{\alpha\in R} B(H_\alpha)$	$S = C^*(G)$, $R = \mathrm{Irrep}(G)$ H_α space of the repr. α
$\delta : S \to M(S\otimes S)$ coassociative, $\kappa : \mathcal{S} \to \mathcal{S}$ $\varepsilon : S \to \mathbb{C}$ co-unit (trivial repr. : $\varepsilon \in R$)	$\delta(U_g) = U_g\otimes U_g$ $\varepsilon(U_g) = 1$, $\kappa(U_g) = U_g^{-1}$
Haar weights h_L, h_R defined on $\mathcal{S}$ $\forall\ a \in p_\alpha S\ h_L(a) = m_\alpha \mathrm{Tr}\ (F_\alpha^{-1} a)$ and $h_R(a) = m_\alpha \mathrm{Tr}\ (F_\alpha a)$ with $F_\alpha \in B(H_\alpha)_+$ st $\mathrm{Tr}\ F_\alpha = \mathrm{Tr}\ F_\alpha^{-1} =: m_\alpha$	$h_L = h_R$ "canonical trace" of $C^*(G)$ $F_\alpha = \mathrm{id}_{H_\alpha}$, $m_\alpha = \dim H_\alpha$
$\Lambda : \mathcal{S} \to H$ GNS construction for h_R $V(\Lambda\otimes\Lambda)(x\otimes y) = (\Lambda\otimes\Lambda)(\delta(x)(1\otimes y))$ $V \in M(\hat{S}_r\otimes S)$ with $\hat{S}_r = (\mathrm{id}\otimes B(H)_*)(V)^-$	$\hat{S}_r = C(G)$, $\hat{\delta}(\hat{a})(g,h) = \hat{a}(gh)$ $\hat{h}$ Haar measure of G (trace)
$\mathcal{F}(a) = (\mathrm{id}\otimes h_R)(V^*(1\otimes a)) \in \hat{S}_r$ for $a \in \mathcal{S}$ $\hat{\mathcal{S}} = \mathcal{F}(\mathcal{S}) \subset \hat{S}_r$, $\hat{h}(\mathcal{F}(a)^*\mathcal{F}(a)) = h_R(a^*a)$	for $a \in p_\alpha S$, $\mathcal{F}(a)$ is a coefficient of α